Derivative Properties for Powers and Polynomials (Section 3.1) **Date:** ______________

We know that the derivative of a function at a point represents a slope and a rate of change. In the last chapter we learned how to estimate values of the derivative of a function given by a graph, a table or a function. Now we learn how to find a formula for the derivative function if we are given a function rule.

We will discover these properties using a graphical approach. Some with the help of our calculator.

I. The Derivative of a Constant Function

1. Let $f(x) = 2$. Graph $f(x)$ and $f'(x)$.

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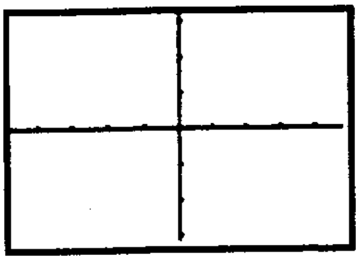

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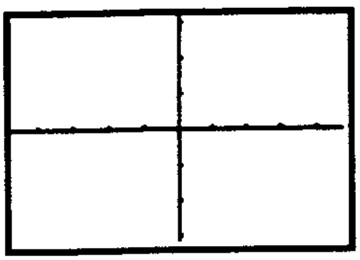

2. Let $f(x) = -1$. Graph $f(x)$ and $f'(x)$.

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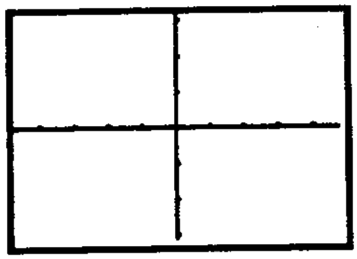

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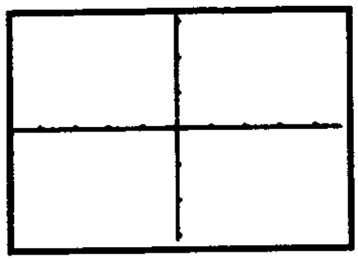

Property: **If k is a number and $f(x) = k$, then $f'(x) =$** ______________

II. The Derivative of a Linear Function

1. Let $f(x) = \frac{1}{2}x - 2$. Graph $f(x)$ and $f'(x)$.

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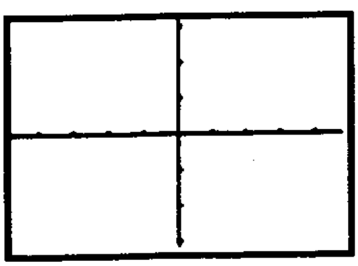

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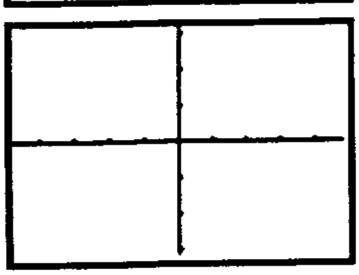

2. Let $f(x) = -3x + 3$. Graph $f(x)$ and $f'(x)$.

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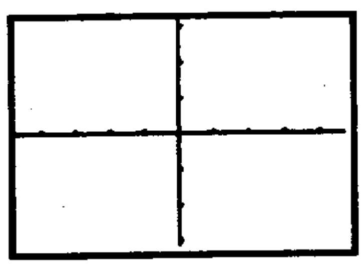

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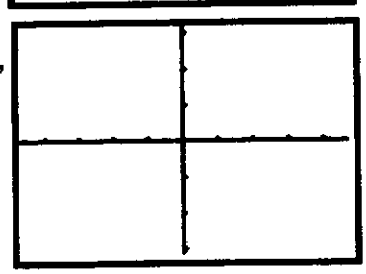

Property: **If $f(x) = mx + b$, then $f'(x) =$** ______________

III. The Derivative of a Power Function ($f(x) = x^n$)

We are going to use our calculator to help us to discover this rule and other rules. For each of the following functions:

a. Set a WINDOW on your calculator of [-4 , 4] by [-15 , 15]
b. Put the given function **f(x)** into **Y1** of your graphing calculator. (You may want to turn it off by deactivating it.)
c. Let **Y2 = nDeriv(Y1,X,X)**.
d. Guess the function that you see in **Y2** and check your guess by putting it into **Y3**.
e. If it matches, record your answer; if it doesn't, try again!
f. Don't forget, we are looking for patterns and generalizations that we can write as a property.

1. $f(x) = x^2$ $\quad f'(x) =$ ____________

2. $f(x) = x^3$ $\quad f'(x) =$ ____________

3. $f(x) = x^4$ $\quad f'(x) =$ ____________

Property: **If $f(x) = x^n$, then $f'(x) =$** ________________ (This is called the **Power Rule.**)

IV. The Derivative of a Constant Times a Function

1. $f(x) = 4x^2$ $\quad f'(x) =$ ______________________

2. $f(x) = -2x^3$ $\quad f'(x) =$ ______________________

3. $f(x) = 7x$ $\quad f'(x) =$ ______________________

Property: **If k is a number and $f(x) = k \cdot g(x)$, then $f'(x) =$** ____________________

V. The Derivatives of Sums and Differences

1. $f(x) = -x^2 - 3x - 6$ $\quad f'(x) =$ ____________________________

2. $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 6,$ $\quad f'(x) =$ ____________________________

Property: **If $f(x) = g(x) + k(x)$, then $f'(x) =$** ____________________________

VI. Question: Does the Power Rule hold for other numbers besides positive whole numbers? How about negative whole numbers? How about rational numbers?

Let's do a **Quick Algebra Review** of exponents first.

Rewrite each expression without negative or rational exponents.

1. 2^{-3} = ______________
2. x^{-n} = ______________
3. $16^{\frac{1}{4}}$ = ______________
4. $x^{\frac{1}{n}}$ = ______________
5. $27^{\frac{4}{3}}$ = ______________
6. $x^{\frac{m}{n}}$ = ______________

1. $f(x)=\dfrac{1}{x}$ = ______________, $f'(x)$ = ______________________________
2. $f(x)=\dfrac{1}{x^2}$ = ______________, $f'(x)$ = ______________________________
3. $f(x)=\sqrt{x}$ = ______________, $f'(x)$ = ______________________________
4. $f(x)=\sqrt[3]{x^2}$ = ______________, $f'(x)$ = ______________________________

VII. Practice Problems

Find the derivative of each function below. (Hint: Rewrite f first using properties of exponents!)

1. $y=2x^4+2+\dfrac{1}{2x^4}$

2. $y=4\sqrt{x}+\dfrac{4}{\sqrt{x}}+\dfrac{\sqrt[4]{x}}{4}$

3. $y=\sqrt{x}\left(x^2-\dfrac{1}{x^2}\right)$

VIII. Using the Derivative Formulas

1. Let $f(x) = -2x^3 + 6x + 8$

   a. Find $f'(2)$

   b. Find $f'(-1)$

   c. Find the equation of the tangent line at $x = -2$.

   d. Find the x-value(s) where the tangent line to the curve is horizontal.

   e. Check your answers to parts a - d above graphically.

2. Earlier in the semester, we looked at the problem where a ball is thrown up in the air. The height, y, of the ball above the ground first increases, and then decreases. It's height t seconds after it is thrown is shown in the chart below.

t (sec)	0	0.5	1	1.5	2	2.5	3	3.5	4
y (feet)	5	36	59	74	81	80	71	54	29

Our objective was to calculate the average rate of change (average velocity) of the ball over an interval and to estimate the instantneous velocity at a specific time. We entered the data into the lists of our calculator, and performed a quadratic regression to find the best fitting quadratic function for the data. The result was:

$$s(t) = -16t^2 + 70t + 5$$

a. Find the velocity function v(t), which will allow us to calculate the velocity of the ball at any time.

$v(t) =$ ______________________________

b. What was the velocity of the ball after 2 seconds? ____________________

c. At what time did the ball reach it's maximum height? $t =$ ____________

d. What is the maximum height of the ball? ____________

e. What was the velocity of the ball when it hit the ground? Solve this algebraically, and check your answer graphically.

$v =$ ____________